

Example 8.2 Wave Propagation in a Rectangle
Equation (8.1.2) is solved in Maple below:

```
> restart:with(inttrans):with(plots):
```

```
>
```

```
> eq:=diff(u(x,t),t)+diff(u(x,t),x);
```

$$eq := \frac{\partial}{\partial t} u(x, t) + \frac{\partial}{\partial x} u(x, t) \quad (1)$$

```
> u(x,0):=1-exp(-x);
```

$$u(x, 0) := 1 - e^{-x} \quad (2)$$

```
> bc1:=u(0,t)=0;
```

$$bc1 := u(0, t) = 0 \quad (3)$$

```
> eqs:=laplace(eq,t,s):
```

```
> eqs:=subs(laplace(u(x,t),t,s)=U(x),eqs);
```

$$eqs := s U(x) - 1 + e^{-x} + \frac{d}{dx} U(x) \quad (4)$$

```
> bc1:=laplace(bc1,t,s):
```

```
> bc1:=subs(laplace(u(0,t),t,s)=U(0),bc1);
```

$$bc1 := U(0) = 0 \quad (5)$$

The solution obtained in the Laplace domain is:

```
> U(x):=rhs(dsolve({eqs,bc1},U(x)));
```

$$U(x) := \frac{e^{-sx}}{s(s-1)} - \frac{-s+1+e^{-x}s}{s(s-1)} \quad (6)$$

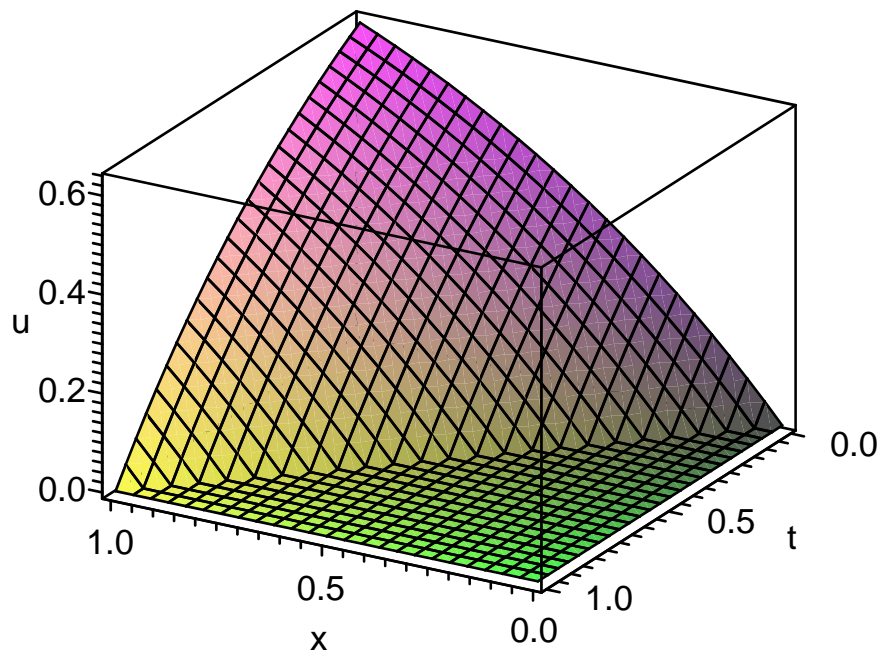
The solution obtained in the time domain is obtained as:

```
> u:=invlaplace(U(x),s,t);
```

$$u := -\text{invlaplace}\left(\frac{e^{-sx}}{s}, s, t\right) + \text{invlaplace}\left(\frac{e^{-sx}}{s-1}, s, t\right) + 1 - e^{-x+t} \quad (7)$$

```
> plot3d(u,x=1e-6..1,t=0..1,axes=boxed,title="Figure Exp. 8.3.",
labels=[x,t,"u"],orientation=[120,60]);
```

Figure Exp. 8.3.



```
> plot([subs(t=0.1,u),subs(t=0.25,u),subs(t=0.5,u),subs(t=1,u)],x=
0..1,axes=boxed,title="Figure Exp. 8.4.",thickness=5,labels=[x,
"u"]);
```

Figure Exp. 8.4.

